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Observer Stabilization of Singularly Perturbed Systems

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Introduction

IN a recent Note, Suzuki and Miura[1] solved the problem of stabilization of singularly perturbed linear systems by linear state feedback. They found that stabilization of the reduced system (fast modes eliminated) would guarantee the stability of the full system. A similar result was obtained by Porter.[2] Such results are useful in designing stable systems where fast and slow modes appear, e.g., systems with parasitic elements or control actuators. However, in most cases the full state vector is unavailable and must be reconstructed from available measurements by a Kalman filter[3] or a Luenberger observer.[4] In this Note, the feedback stabilization is developed for singularly perturbed systems where the state vector is reconstructed by a Luenberger observer based on the reduced system. It is shown, by the lemma of Klimushchev and Krasovskii,[5,10] that with this observer feedback the full system remains stable. To illustrate the use of the method, the results are specialized to systems with parasitic elements and to autopilot stabilization of systems with control actuators.

Main Result

The singularly perturbed systems under consideration have the following form

$$\dot{X}_1 = A_{11}X_1 + A_{12}X_2 + B_1 u \tag{1a}$$

$$\epsilon\dot{X}_2 = A_{21}X_1 + A_{22}X_2 + B_2 u \tag{1b}$$

$$y = C_1 X_2 + C_2 X_2 + Du \tag{1c}$$

where X_1 represents the state vector of the slow modes and X_2 represents that of the fast modes. The output vector y represents the available measured information and its entries are linear combinations of the internal states X_1 and X_2 and the control vector u. The parameter ϵ is nonnegative and represents the response time of the fast modes of the system. The set of Eq. (1) will be referred to as the *full system*.

Assume that A_{22} is stable (i.e., its spectrum is in the open left half-plane). The *reduced system* is obtained by ignoring the fast modes (i.e., setting $\epsilon = 0$) and this yields

$$\dot{\bar{X}}_1 = \bar{A}_{11}\bar{X}_1 + \bar{B}_1 u \tag{2a}$$

$$\bar{y} = \bar{C}_1 \bar{X}_1 + \bar{D}u \tag{2b}$$

where

$$\bar{A}_{11} = A_{11} - A_{12}A_{22}^{-1}A_{21}$$

$$\bar{B}_1 = B_1 - A_{12}A_{22}^{-1}B_2$$

$$\bar{C}_1 = C_1 - C_2A_{22}^{-1}A_{21}$$

$$\bar{D} = D - C_2A_{22}^{-1}B_2$$

Suppose stabilization is obtained by state feedback from a Luenberger observer[4] designed for the reduced system (2) where the fast modes are ignored. This would be a practical approach for most deterministic systems where a feedback compensator was desired. The observer is a dynamic system:

$$\dot{\hat{X}}_1 = \bar{A}_{11}\hat{X}_1 + \bar{B}_1 u + K(y - \hat{y}) \tag{3a}$$

$$\hat{y} = \bar{C}_1\hat{X}_1 + \bar{D}u \tag{3b}$$

and the stabilizing control is

$$u = G\hat{X}_1 \tag{4}$$

This produces a feedback compensator of the form:

$$\dot{\hat{X}}_1 = [\bar{A}_{11} + \bar{B}_1 G - K\bar{C}_1 - K\bar{D}G]\hat{X}_1 + Ky \tag{5a}$$

$$u = G\hat{X}_1 \tag{5b}$$

which accepts the system output y and produces the stabilizing control u.

Set $e_1 = \hat{X}_1 - X_1$ and use Eqs. (1, 3, and 4) to obtain

$$\dot{e}_1 = [\bar{A}_{11} - K\bar{C}_1 - (A_{12} - KC_2)A_{22}^{-1}B_2G]e_1$$
$$- (A_{12} - KC_2)A_{22}^{-1}(A_{21} + B_2G)X_1 - (A_{12} - KC_2)X_2 \tag{6}$$

Also use Eq. (4) in Eq. (1) to produce

$$\dot{X}_1 = (A_{11} + B_1G)X_1 + A_{12}X_2 + B_1Ge_1 \tag{7a}$$

$$\epsilon\dot{X}_1 = (A_{21} + B_2G)X_1 + A_{22}X_2 + B_2Ge_1 \tag{7b}$$

Let $Z = (X_1, e_1)^T$ and rewrite Eqs. (6) and (7) as

$$\dot{Z} = H_{11}Z + H_{12}X_2 \tag{8a}$$

$$\epsilon\dot{X}_2 = H_{21}Z + H_{22}X_2 \tag{8b}$$

where

$$H_{11} = \begin{bmatrix} A_{11} + B_1G & B_1G \\ -(A_{12} - KC_2)A_{22}^{-1}(A_{21} + B_2G) & T_{11} \end{bmatrix}$$

$$T_{11} = \bar{A}_{11} - K\bar{C}_1 - (A_{12} - KC_2)A_{22}^{-1}B_2G$$

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$$H_{12}=\begin{bmatrix} A_{12} \\ -(A_{12}-KC_2) \end{bmatrix}$$

$$H_{21}=[A_{21}+B_2G \qquad B_2G]$$

$$H_{22}=A_{22}$$

The result of Klimushchev and Krasovakii[5] can be applied to the system in Eqs. (8).

$K-K$ Lemma

Given $\dot{w}_1=H_{11}w_1+H_{12}w_2$ and $\epsilon\dot{w}_2=H_{21}w_1+H_{22}w_2$, there exists $\epsilon_1>0$ such that this system is stable for any ϵ in $[0, \epsilon_1]$ when H_{22} and $\bar{H}_{11}=H_{11}-H_{12}H_{22}^{-1}H_{21}$ are stable.

Since $H_{22}=A_{22}$ and is assumed stable, consider $\bar{H}_{11}$ which is the system matrix for the reduced version ($\epsilon=0$) of Eq. (8) and hence

$$\bar{H}_{11}=\begin{bmatrix} \bar{A}_{11}+\bar{B}_1G & \bar{B}_1G \\ 0 & \bar{A}_{11}-K\bar{C}_1 \end{bmatrix} \tag{9}$$

which is stable if the gains G and K can be chosen to make $\bar{A}_{11}+\bar{B}_1G$ and $\bar{A}_{11}-K\bar{C}_1$ stable. It is well known[6] that this is possible if $(\bar{A}_{11},\bar{B}_1,\bar{C}_1)$ is controllable and observable. This is summarized as follows.

Theorem: if the reduced system (2) is controllable and observable, then there exists a positive ϵ_1 such that the compensator gains G and K can be chosen so that, for any ϵ in $[0, \epsilon_1]$, the full system (1), with observer feedback (4) from a reduced-order observer (3), will be stablized and the observer estimate will converge to the actual state of the slow system.

This means that a reduced-order observer can generate the proper feedback to stabilize a singularly pertrubed system for small ϵ. Such an approach produces a greatly simplified feedback compensator and makes it possible for the designer to ignore the fast modes and still have a system with sufficient robustness to remain stable. Decay rates can be approximately determined from Eq. (9) but better approximations may be possible with the methods outlined in Kokotovic[7] applied to Eqs. (6) and (7).

Applications

To illustrate this design philosophy, the singular perturbations result will be applied to two cases of interest: parasitic elements and autopilot-actuator stabilization.

Parasitic Elements

In this case the system X_1 is a dynamic system with small parasitic elements in the form of stray capacitances and lead inductances represented by system X_2 (with $B_2=0$ and $C_2=0$). The undriven parasitic elements would be stable (i.e. A_{22} stable) because dissipation would tend to damp parasitic oscillations. The feedback compensator to stabilize the full system would be much too complex to implement; however, due to the small size of the stray effects, the time constants for the parasitic terms would be small and hence ϵ would be small. Consequently, the approach described earlier can be used to design a reduced-order compensator without sacrificing stability as long as $(\bar{A}_{11}, B_1, C_1)$ is controllable and observable. Some estimate of the parameters A_{12},A_{22} of the parasitic system must be available in order to use the approach effectively; the stray effects cannot simply be ignored. (See Shensa[8] and Desoer and Shensa[9] for further discussion of parasitics.)

Autopilot—Actuator Stabilization

In this case, the slow system represents the air-frame dynamics of a vehicle and is given by

$$\dot{X}_c=A_cX_c+B_c\delta \tag{10a}$$

$$y=C_cX_c+D_c\delta \tag{10b}$$

The control law δ will be generated by an autopilot (feedback compensator) which senses deviations y from a nominal flight path and produces correction commands to stabilize the flight and maintain the course. These commands are not applied directly to the control surfaces but through actuators, such as electric motors or hydraulic valves and pistons. The actuator subsystem is given by

$$\epsilon\dot{X}_a=A_aX_a+B_au \tag{11a}$$

$$\delta=MX_a \tag{11b}$$

where A_a can be designed to be stable, and the actuator system will have a fast response (i.e., ϵ will be small) compared to the much slower response of the vehicle. For practical reasons, when $\epsilon=0$, the actuator must operate as a unit gain; this means

$$-MA_a^{-1}B_a=I \tag{12}$$

The autopilot commands u can be generated by a feedback compensator which accepts the measured deviations y and produces a desired correction u. These corrections drive the actuator subsystem [Eq. (11)] which in turn guides the vehicle [Eq. (10)]. To take account of vehicle and actuator dynamics would require a high-order autopilot, but the singular perturbations method can be applied to obtain a reduced-order autopilot of the form given by Eq. (5). This can be done because systems (10) and (11) have the form of Eq. (1) where $X_1=X_e$ and $X_2=X_a$ and $A_{11}=A_c$, $A_{12}=B_cM$, $A_{21}=0$, $A_{22}=A_a$, $B_1=0$, $B_2=B_a$, $C_1=C_c$, $C_2=D_c\ M$, $D=0$. From this the reduced-order system has the form of Eq. (2) with [using Eq. (12)]

$$\bar{A}_{11}=A_c$$

$$\bar{B}_1=-B_cMA_a^{-1}B_a=B_c$$

$$\bar{C}_1=C_c$$

$$\bar{D}=-D_cMA_a^{-1}B_a=D_c$$

In other words, the reduced-order system in this case is the vehicle dynamics without the actuator. It is reasonable to assume that the vehicle is a controllable and observable system. Hence, the autopilot is easily seen to be

$$\dot{\hat{X}}_1=[A_c+B_cG-KC_c-KD_cG]X+Ky \tag{13a}$$

$$u=G\hat{X}_1 \tag{13b}$$

where K is chosen to make the autopilot respond quickly (i.e., A_c-KC_c has rapid decay) and G is chosen to keep the vehicle on a stable flight path (i.e., A_c+B_cG stable). Depending on the particular application, the choice of G may be done by pole placement techniques or by optimal regulator theory.

The singular perturbations result guarantees that the composite system is sufficiently robust to remain stable and for the autopilot to respond quickly. It should be clear that this result justifies the often-used design philosophy of neglecting fast actuator dynamics in autopilot stabliization.

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Unusual Maneuvers on Nimbus and Landsat Spacecraft

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Introduction

THE current version of the attitude control system used for both the Nimbus and Landsat series of spacecraft was first developed for use on the Nimbus 4 spacecraft. This control system has been used on the following five spacecraft now in orbit:

Spacecraft	Launch date
Nimbus 4	April 8, 1970
Nimbus 5	Dec. 11, 1972
Nimbus 6	June 12, 1975
Landsat 1	July 23, 1972
Landsat 2	Jan. 22, 1975

Despite a design life ranging from six months for Nimbus 4 to one year for Nimbus 6, all five spacecraft are still operational. Orbital life times range from almost two years to seven years.

This record of reliability was achieved despite several component failures and operational mishaps. One of the reasons this was possible is that the large number of commands and telemetry functions available provide great flexibility. Maneuvers have been devised and performed that were never thought of when the control system was designed. Several examples of such maneuvers are presented in this paper. These include a yaw turn-around maneuver using a failed gyro, elimination of cold gas thrusting to unload momentum, and adjusting the orbit without using the Orbit Adjust Subsystem.

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Attitude Control System Description

The Nimbus/Landsat Attitude Control System (ACS) is a three-axis zero-momentum system that orients the spacecraft to the local vertical (pitch and roll) and the orbital plane (yaw). Reaction wheels provide normal control torques, and cold-gas thrusters are provided for momentum unloading. Infrared horizon scanners provide pitch and roll error signals. An inertial quality gyro is used in a gyro-compassing loop to provide yaw error signals.

The Nimbus 4 ACS has two features not provided on the other spacecraft. These are 1) a sun sensor to provide yaw error signals in the event of a gyro failure, and 2) a gravity gradient rod to be used in an auxiliary mode in which gravity gradient torques are used to unload momentum. The spacecraft operate in low altitude (920 to 1110 km) circular, near-polar sun-synchronous orbits. Orbital periods are 103 to 107 min, of which approximately one-third is spent in the Earth's umbra.

Yaw-Around Maneuver with Failed Gyro

After one year in orbit the Nimbus 4 gyro spin motor failed. Yaw control was switched to the yaw sun sensor system. This system consists of two sensors, one facing forward and one facing aft. Both sensors are connected to provide stable nulls when facing the sun. This provision allows the spacecraft to fly always in the forward position. This is desirable for two reasons: 1) The Image Dissector Camera Subsystem includes a provision for image motion compensation; the images are distorted when the spacecraft flies backwards, and 2) solar arrays are driven by a fixed rate bias signal plus a sun sensor signal; when the spacecraft flies backward the tracking errors are larger in the sunlight, and the direction of rotation is incorrect in the umbra, leading to a large tracking error at sunrise.

However, with a stable null on both yaw sun sensors the spacecraft can fly either forward or backward. As the

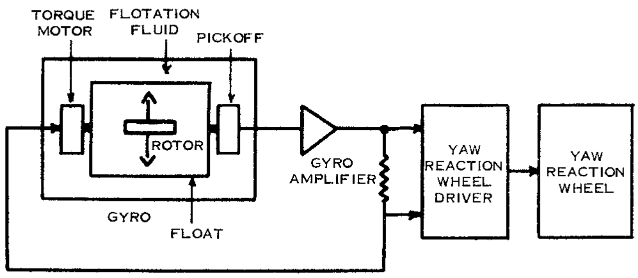

Fig. 1 Gyro circuit block diagram.

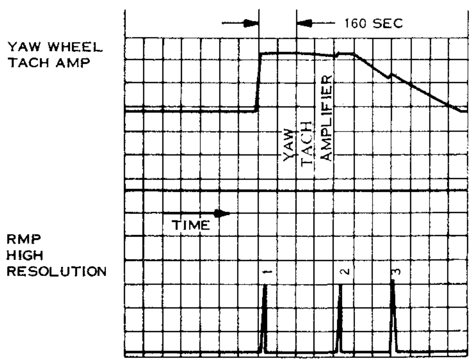

Fig. 2 Telemetry records during turn-around maneuver.